ERRATA TO *EQUATIONS, INEQUATIONS AND INEQUALITIES CHARACTERIZING THE CONFIGURATIONS OF TWO REAL PROJECTIVE CONICS*

EMMANUEL BRIAND

These are corrections for both versions of the paper *Equations, inequations and inequalities characterizing the configurations of two real projective conics*:

- Version published in *Applicable Algebra in Engineering, Communication and Computing*, vol. 18 (1-2), pp. 21-52 (`https://dx.doi.org/10.1007/s00200-006-0023-8`).
- Preliminary version published on arxiv: arXiv:math/0505628 [math.AC] (`https://doi.org/10.48550/arXiv.math/0505628`))

There, Proposition 8 is incorrect, as pointed out by Sylvain Petitjean.

Here the error is fixed. There error has two sources: a mistake in Figure 6 and some wrong calculations in section 3.4.1.

1. In Figure 6

The first mistake occurs in Figure 6: in the representation of the pencil of type III, the arrows (pointing towards the inner conics) should be reverted.

2. Section 3.4.1 corrected

Some computations in section 3.4.1 of the paper are incorrect. Below there is a corrected version with some additional details to make the reasoning clearer.

The *antisymmetric invariant* is

$$\mathcal{A} = \Phi_{30}\Phi_{12}^3 - \Phi_{03}\Phi_{21}^3.$$

First it is homogeneous of even degree, 6, in f, as well as in g. So its sign depends only on the algebraic conics, not on the quadratic forms defining them.

Choose f_0 and g_0 as in Table I for one of the orbits of pencils Ia, II, IIa, III, IIIa or V. Set

$$f = f_0 + t_1 g_0, \qquad g = f_0 + t_2 g_0. \tag{1}$$

Observe that if the pencil generated by f_0 and g_0 is represented as in Figures 5 or 6, then $[f_0]$ is always the

Date: November 2007.

singular conic represented at the bottom of the drawing, and $[g_0]$ the singular conic represented at its top, except for V, where the singular conic, at the bottom, is $[g_0]$.

As a consequence, when $\{[f],[g]\}$ lies in Class IaN, IIN, IIaN or IIIaN, the inner conic is the one nearer from f_0 (since the arrows pointing to the inner conics in Figures 5 and 6 go from $[g_0]$ towards $[f_0]$), that is the one whose parameter (t_1 or t_2) has smaller absolute value. When $\{[f],[g]\}$ lies in Class IIIN , then the inner conic is the one whose parameter has greater absolute value (since in this case the arrows go from $[f_0]$ towards $[g_0]$).

When $\{[f],[g]\}$ lies in Class VN, one determines that $[f]$ lies inside $[g]$ if and only if $t_1 < t_2$ (for instance by considering the point $(1:0:-t_1)$ that is on $[f]$ and not on $[g]$, and checking that it lies inside $[g]$ if and only if $t_1 < t_2$).

Evaluate the antisymmetric invariant on (f,g). This yields the following values:

Orbit of Pencil	$\mathcal{A}(f,g)$
Ia	$-(t_1-t_2)^2(t_1^2-t_2^2)\left((t_1t_2-12)^2-4(t_1+t_2)^2\right)/256$
II	$-t_1^2t_2^2(t_1-t_2)^2(t_1^2-t_2^2)/256$
IIa	$-t_1^2t_2^2(t_1-t_2)^2(t_1^2-t_2^2)/256$
III	$(t_1-t_2)^2(t_1^2-t_2^2)/256$
IIIa	$(t_1-t_2)^2(t_1^2-t_2^2)$
V	0

For II, IIa, III, IIIa, the sign of $\mathcal{A}(f,g)$ is clear.This proves the following proposition.

Proposition 8. *Let $([f],[g])$ be a couple of distinct proper non-empty conics.*

- *If $\{[f],[g]\}$ is in Class IIN, IIaN or IIIN then $[f=0]$ lies inside*[1] *of $[g=0]$ if and only if $\mathcal{A}(f,g)>0$.*
- *If $\{[f],[g]\}$ is in Class IIIaN then $[f=0]$ lies inside of $[g=0]$ if and only if $\mathcal{A}(f,g)<0$.*

For IaN and VN the relative position of $[f]$ and $[g]$ can't be deduced from the sign of $\mathcal{A}(f,g)$. We need other methods to solve the question in these two cases.

3. Further corrections

After correcting section 3.4.1, there remain only a few details to be settled:

[1]only at the neighborhood of the double intersection point for class IIN.

- In section 3.4.2, the actual value of $\mathcal{B}(xz - y^2 + t_1\,x^2, xz - y^2 + t_2\,x^2)$ is $3(t_1 - t_2)x^2/4$. The factor 3 is missing in the paper (this mistake has no consequences, since it has no influence on the sign of $\mathcal{B}$).
- In section 3.5.2, last paragraph, the first item
  - IIN, IIaN, IIIN, IIIaN: $\mathcal{A} < 0$

  should be changed into:
  - IIN, IIaN, IIIN: $\mathcal{A} > 0$.
  - IIIaN: $\mathcal{A} < 0$.

Finally, Figure 7 in example 4.3 happens to be correct despite the mistake in Proposition 8.

Email address: ebriand@us.es
URL: http://emmanuel.jean.briand.free.fr/